# Optimal Design of Autonomous Vehicle Zones in Transportation Networks (17-01059)

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## 1. INTRODUCTION

Autonomous vehicles (AVs) are expected to offer extraordinary improvements to both the safety and efficiency of existing roadways and mobility systems. Although it will be many years before a widespread adoption of the AV technology, recent developments suggest that they are on the horizon. To leverage the growing adoption of AVs, government agencies may dedicate certain traffic lanes, highway segments or even areas of networks to AVs only to facilitate the formulation of vehicle platoons to further improve throughput. Subsequently implemented are innovative control strategies that aim to achieve system optimum in those areas. The dedicated AV areas will expand gradually as the level of market penetration of AVs increases and eventually support a fully connected and automated mobility in the whole system.

This paper deals with a particular issue in the above infrastructure adaptation planning process and aims to present a mathematical framework for optimal design of AV zones in a general network with both conventional vehicles (CVs) and AVs.

## 2. PROBLEM DESCRIPTION

We consider a network where both AVs and CVs are present. The origin-destination (O-D) matrices of vehicular trips of AVs and CVs are given. It is envisioned that a government agency strategically designs AV zones on a road network.

### AV Zone

- An area consists of a set of links that are tailored to AVs
- It is cordoned off by a virtual loop

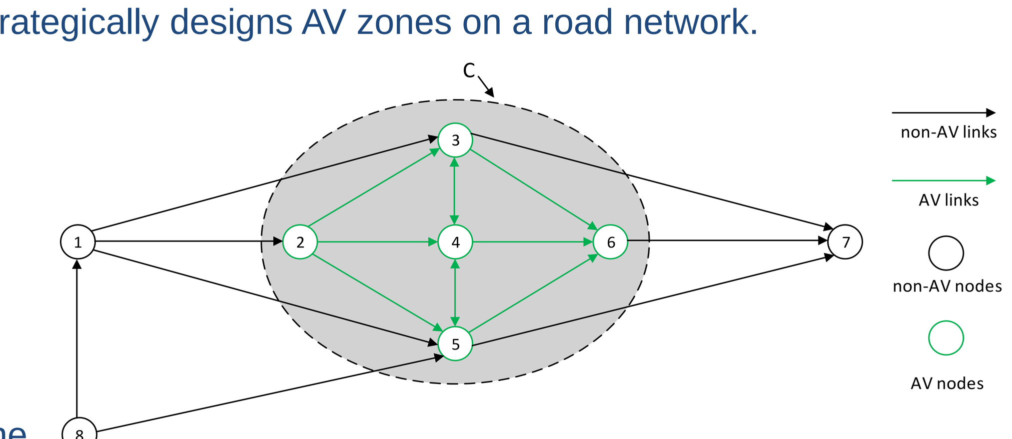


FIGURE 1 An example of AV zone

### Operational Concept

- Only AVs are allowed to use the AV links
- When entering the zone, AVs must report their exits of the zone to the control center, which routes AVs to traverse the zone
- Based on AVs' entrances and exits, the control center routes AVs to minimize the total travel time in the zone

In the presence of an AV zone, when making their route choices,

- CVs need to avoid the zone
- AVs will decide whether to access the zone, and where to enter and exit

### Basic Assumptions

i. We assume that AVs perceive their travel times to be the minimum travel times between their corresponding entrances and exits of the AV zone.
ii. All vehicles are assumed to minimize their own perceived trip times.
iii. The per-lane capacity of links within the AV zone is much larger than those of regular links due to vehicle automation.
iv. The capacity of a regular link with mixed traffic of CVs and AVs remains the same as when only CVs use the link.
v. The performance functions of regular and AV links may be different, but all are increasing functions with link flows.
vi. In the network equilibrium model, there exists at least one usable path between each O-D pair for both AVs and CVs.

## 3. MIXED ROUTING EQUILIBRIUM

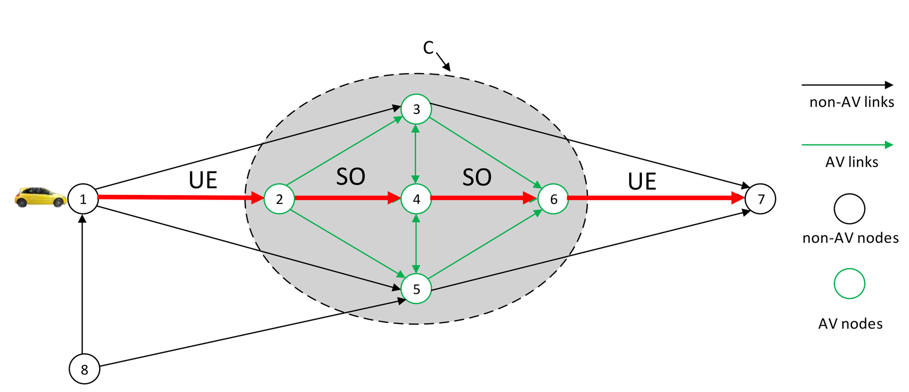


FIGURE 2 A routing plan

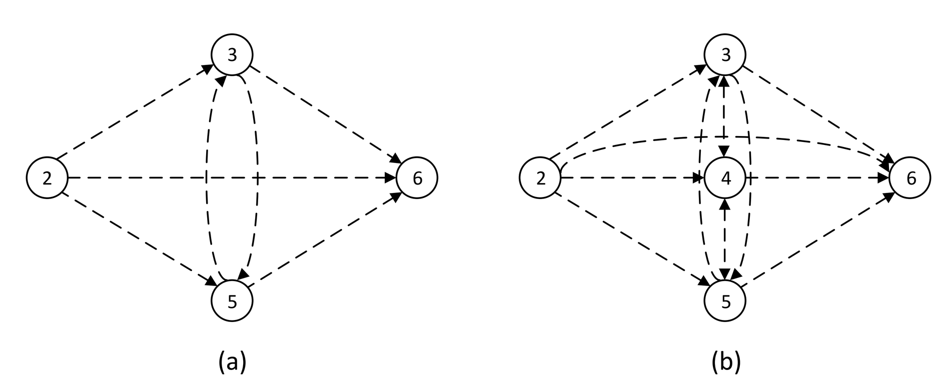


FIGURE 3 Dummy AV networks

### Travel Time of Dummy Links

With a given traffic flow distribution of the AV network, $v_{\tilde{a}}, \forall \tilde{a} \in \tilde{A}$, finding the shortest path can be formulated as follows for each E-E pair $\widetilde{w} \in \widetilde{W}$:

**SP:**

$$\min_{\boldsymbol{z}} \sum_{\tilde{a} \in \tilde{A}} t_{\tilde{a}}(v_{\tilde{a}}) z_{\tilde{a}}^{\widetilde{w}}$$

$$\text{s.t. } \tilde{\Delta} \boldsymbol{z}^{\widetilde{w}} = \tilde{E}^{\widetilde{w}} \quad (1)$$

$$z_{\tilde{a}}^{\widetilde{w}} \geq 0 \qquad \forall \tilde{a} \in \tilde{A} \quad (2)$$

Optimality conditions:

(1)-(2)

$$\left[t_{\tilde{a}}(v_{\tilde{a}}) - \kappa_i^{\widetilde{w}} + \kappa_j^{\widetilde{w}}\right] z_{\tilde{a}}^{\widetilde{w}} = 0 \qquad \forall \tilde{a} = (i,j) \in \tilde{A}, \widetilde{w} \in \widetilde{W} \quad (4)$$

$$t_{\tilde{a}}(v_{\tilde{a}}) - \kappa_i^{\widetilde{w}} + \kappa_j^{\widetilde{w}} \geq 0 \qquad \forall \tilde{a} = (i,j) \in \tilde{A}, \widetilde{w} \in \widetilde{W} \quad (5)$$

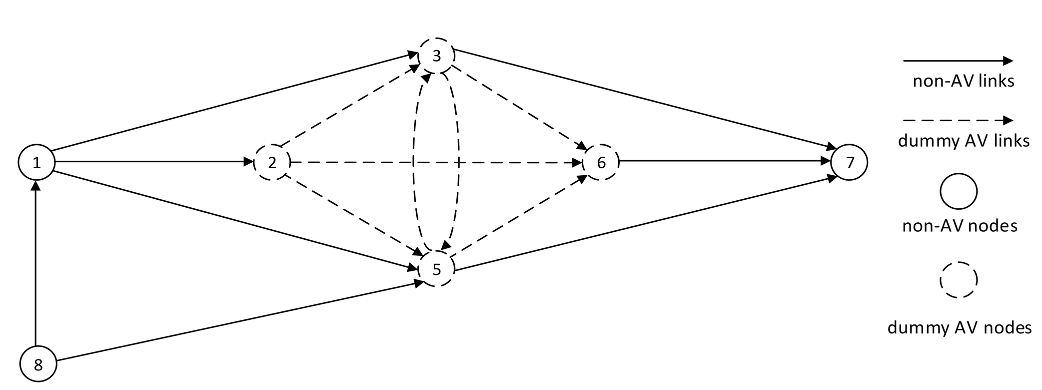


FIGURE 4 A revised network

The travel time of dummy AV link $a \in \hat{A}$ (denoted as $c_a$):

$$c_a = \sum_{\widetilde{w} \in \widetilde{W}} \beta_a^{\widetilde{w}} \left[ \sum_{\tilde{a} \in \tilde{A}} t_{\tilde{a}}(v_{\tilde{a}}) z_{\tilde{a}}^{\widetilde{w}*} \right] \quad (3)$$

### User Equilibrium in the Revised Network

$$\boldsymbol{\Delta x}^{w,m} = \boldsymbol{E}^{w,m} d^{w,m} \qquad \forall m \in M, w \in W \quad (6)$$

$$x_a^{w,A} \geq 0 \qquad \forall a \in A \cup \hat{A}, w \in W \quad (7)$$

$$x_a^{w,C} \geq 0 \qquad \forall a \in A, w \in W \quad (8)$$

$$x_a^{w,C} = 0 \qquad \forall a \in \hat{A}, w \in W \quad (9)$$

$$v_a = \sum_{w \in W} \sum_{m \in M} x_a^{w,m} \qquad \forall a \in A \cup \hat{A} \quad (10)$$

$$t_a(v_a) - \rho_i^{w,m} + \rho_j^{w,m} \geq 0 \qquad \forall a = (i,j) \in A, w \in W, m \in M \quad (11)$$

$$c_a(v_a) - \rho_i^{w,A} + \rho_j^{w,A} \geq 0 \qquad \forall a = (i,j) \in \hat{A}, w \in W \quad (12)$$

$$\left(t_a(v_a) - \rho_i^{w,m} + \rho_j^{w,m}\right) \cdot x_a^{w,m} = 0 \qquad \forall a = (i,j) \in A, w \in W, m \in M \quad (13)$$

$$\left(c_a(v_a) - \rho_i^{w,A} + \rho_j^{w,A}\right) \cdot x_a^{w,A} = 0 \qquad \forall a = (i,j) \in \hat{A}, w \in W \quad (14)$$

### System-Optimum Routing within the AV Network

$$\sum_j x_{o(\widetilde{w}),j}^{\widetilde{w}} - \sum_k x_{k,o(\widetilde{w})}^{\widetilde{w}} = \sum_{a \in \hat{A}} \beta_a^{\widetilde{w}} \sum_{w \in W} x_a^{w,A} \qquad \forall \widetilde{w} \in \widetilde{W} \quad (15)$$

$$\sum_j x_{d(\widetilde{w}),j}^{\widetilde{w}} - \sum_k x_{k,d(\widetilde{w})}^{\widetilde{w}} = -\sum_{a \in \hat{A}} \beta_a^{\widetilde{w}} \sum_{w \in W} x_a^{w,A} \qquad \forall \widetilde{w} \in \widetilde{W} \quad (16)$$

$$\sum_j x_{i,j}^{\widetilde{w}} - \sum_k x_{k,i}^{\widetilde{w}} = 0 \qquad \forall i \in \widetilde{N} \backslash \{o(\widetilde{w}), d(\widetilde{w})\}, \widetilde{w} \in \widetilde{W} \quad (17)$$

$$x_{\tilde{a}}^{\widetilde{w}} \geq 0 \qquad \forall \tilde{a} \in \tilde{A}, \widetilde{w} \in \widetilde{W} \quad (18)$$

$$v_{\tilde{a}} = \sum_{\widetilde{w} \in \widetilde{W}} x_{\tilde{a}}^{\widetilde{w}} \qquad \forall \tilde{a} \in \tilde{A} \quad (19)$$

$$t_{\tilde{a}}(v_{\tilde{a}}) + v_{\tilde{a}} t'_{\tilde{a}}(v_{\tilde{a}}) - \tilde{\rho}_i^{\widetilde{w}} + \tilde{\rho}_j^{\widetilde{w}} \geq 0 \qquad \forall \tilde{a} \in \tilde{A}, \widetilde{w} \in \widetilde{W} \quad (20)$$

$$\left(t_{\tilde{a}}(v_{\tilde{a}}) + v_{\tilde{a}} t'_{\tilde{a}}(v_{\tilde{a}}) - \tilde{\rho}_i^{\widetilde{w}} + \tilde{\rho}_j^{\widetilde{w}}\right) \cdot x_{\tilde{a}}^{\widetilde{w}} = 0 \qquad \forall \tilde{a} \in \tilde{A}, \widetilde{w} \in \widetilde{W} \quad (21)$$

**Definition:** At the mixed routing equilibrium, for the same mode, *perceived* travel times of utilized paths between an O-D pair are the same, but less than or equal to that of any unutilized usable path between the same O-D pair.

Mathematically, we can define the mixed routing equilibrium conditions (MRE) for the original network as (1)-(21).

**Proposition:** MRE has at least one solution. However, even if all the link performance functions of both the regular and AV links are strictly monotone, we cannot guarantee the uniqueness of the link flow solution to MRE, as the travel time functions of dummy links may not be strictly monotone with respect to the link flows in the revised network.

## 4. OPTIMAL DESIGN OF AUTONOMOUS VEHICLE ZONE

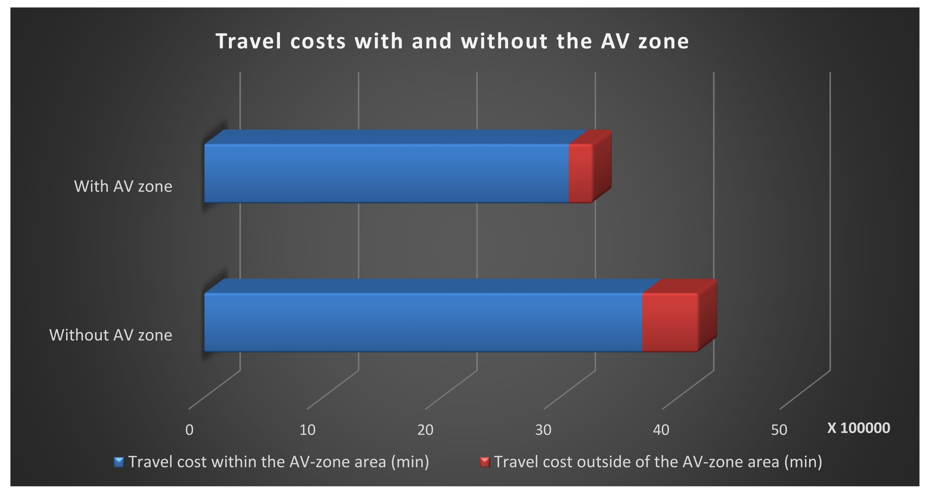
### Mixed-integer bi-level programming model

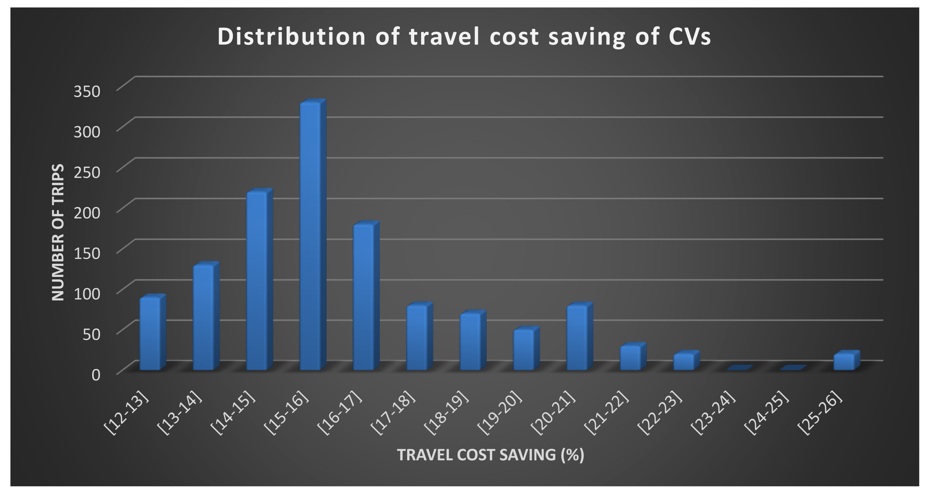
- Lower-level: the proposed mixed routing equilibrium model
- Upper-level: where to set up the AV zone, i.e., which links are upgraded to be AV links.

### A Tailored Simulated Annealing Algorithm

- Its basic idea is to consider a neighboring solution of the current solution at each step, and apply a probability function to decide whether to move to the new solution or not.

### Numerical Example



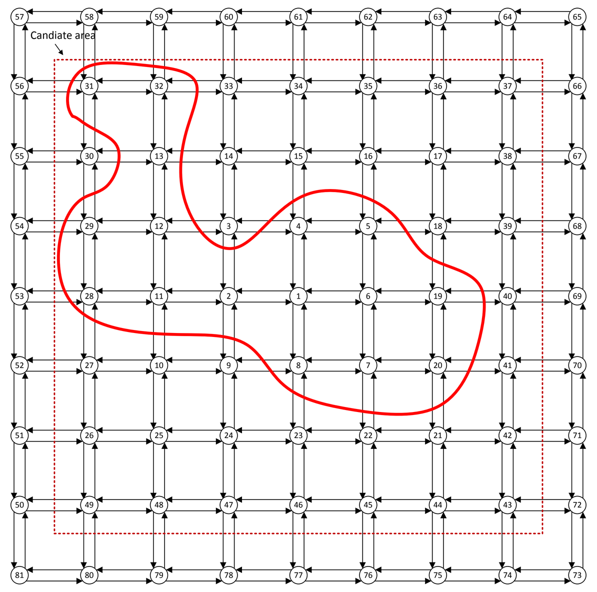



FIGURE 5 Optimal AV zone design

### Numerical Example

| O-D | CV | AV |
|---|---|---|
| 1-7 | 40 | 30 |
| 8-7 | 25 | 15 |

TABLE 1 Equilibrium link flow for the original network

| Link | CV flow | AV flow | Travel time (min) | Link | CV flow | AV flow | Travel time (min) |
|---|---|---|---|---|---|---|---|
| 1-2 | 0.00 | 34.06 | 35.06 | 4-3 | 0.00 | 9.32 | 3.61 |
| 1-3 | 23.96 | 0.00 | 73.87 | 4-5 | 0.00 | 0.00 | 1.00 |
| 1-5 | 28.81 | 7.84 | 37.65 | 4-6 | 0.00 | 5.16 | 5.30 |
| 2-3 | 0.00 | 15.48 | 7.16 | 5-4 | 0.00 | 0.65 | 1.22 |
| 2-4 | 0.00 | 13.82 | 3.30 | 5-6 | 0.00 | 15.05 | 6.02 |
| 2-5 | 0.00 | 4.76 | 2.59 | 5-7 | 41.04 | 0.00 | 84.09 |
| 3-4 | 0.00 | 0.00 | 1.50 | 6-7 | 0.00 | 22.09 | 46.18 |
| 3-6 | 0.00 | 1.88 | 1.94 | 8-1 | 12.76 | 11.91 | 25.67 |
| 3-7 | 23.96 | 22.91 | 47.87 | 8-5 | 12.24 | 3.09 | 63.32 |

TABLE 2 Equilibrium link flow for the dummy network

| Link | AV flow | Travel time (min) | Link | AV flow | Travel time (min) |
|---|---|---|---|---|---|
| 2-3 | 22.91 | 6.91 | 3-6 | 0.00 | 1.94 |
| 2-5 | 0.32 | 2.59 | 5-3 | 0.00 | 4.82 |
| 2-6 | 10.83 | 8.60 | 5-6 | 11.25 | 6.02 |
| 3-5 | 0.00 | 2.50 | | | |

TABLE 3 System-optimum path flow pattern within the AV zone

| E-E | Path | Path flow | Path travel time (min) | Marginal path travel time (min) |
|---|---|---|---|---|
| 2-3 | 2-3 | 13.60 | 7.16 | 12.32 |
| | 2-4-3 | 9.32 | **6.91** | 12.32 |
| | 2-5-4-3 | 0.00 | 7.41 | 12.32 |
| 2-5 | 2-5 | 0.32 | **2.59** | 4.17 |
| | 2-3-4-5 | 0.00 | 9.66 | 14.82 |
| | 2-4-5 | 0.00 | 4.30 | 6.61 |
| 2-6 | 2-3-4-6 | 0.00 | 13.96 | 23.42 |
| | 2-3-4-5-6 | 0.00 | 15.67 | 25.85 |
| | 2-3-6 | 1.88 | 9.10 | 15.20 |
| | 2-4-3-6 | 0.00 | 8.85 | 15.20 |
| | 2-4-5-6 | 0.00 | 10.32 | 17.64 |
| | 2-4-6 | 4.51 | **8.60** | 15.20 |
| | 2-5-4-3-6 | 0.00 | 9.35 | 15.20 |
| | 2-5-4-6 | 0.65 | 9.10 | 15.20 |
| | 2-5-6 | 3.79 | **8.60** | 15.20 |
| 3-5 | 3-4-5 | 0.00 | **2.50** | 2.50 |
| 3-6 | 3-4-5-6 | 0.00 | 8.52 | 13.53 |
| | 3-4-6 | 0.00 | 6.80 | 11.10 |
| | 3-6 | 0.00 | **1.94** | 2.88 |
| 5-3 | 5-4-3 | 0.00 | **4.82** | 8.15 |
| 5-6 | 5-4-3-6 | 0.00 | 6.77 | 11.03 |
| | 5-4-6 | 0.00 | 6.52 | 11.03 |
| | 5-6 | 11.25 | **6.02** | 11.03 |

TABLE 4 Perceived travel times with and without the AV zone

| Scenario | O-D | Perceived travel time of CV (min) | Perceived travel time of AV (min) |
|---|---|---|---|
| Without AV zone | 1-7 | 110.88 | 110.88 |
| | 1-8 | 136.04 | 136.04 |
| With AV zone | 1-7 | 121.74 | 89.84 |
| | 1-8 | 147.40 | 115.51 |

TABLE 4 System and AV-zone area travel times with and without the AV zone

| Scenario | System travel time (min) | Travel time within the AV-zone area (min) |
|---|---|---|
| Without AV zone | 13,202.75 | 1,193.09 |
| With AV zone | 12,987.27 | 324.69 |